Methods of constructing perspective polygons

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Abstract : A new property of conics is demonstrated and the methods of constructing perspective polygons are revealed by means of joining the intersections between conics and the pencils from a common point which is the intersection of the symmetric line and the line passing through the coincided axes of two identical and symmetrical conics regardless of the number and direction of pencils from that point. Dual of this new property of conics is also demonstrated. Theorems can be proved by AutoCAD or MathLab. This type of proving is numerical proof. Proving by deductive and analytical methods are still unsolved.

Theorem 1

To demonstrate that the intersections of cords obtaining from intersections of the same couple of pencils from the intersection of the symmetry line and the line passing through the coincided axes of two identical and symmetrical conics will be perspective from the symmetry line regardless of the number and direction of pencils from that point.

Figure 1 shows two perspective polygons PQRST and P'Q'R'S'T' which are perspective from the line of symmetry (line x) which intersects with the line passing through the coincided axes of two identical conics at point A regardless of the number and direction of pencils from A.

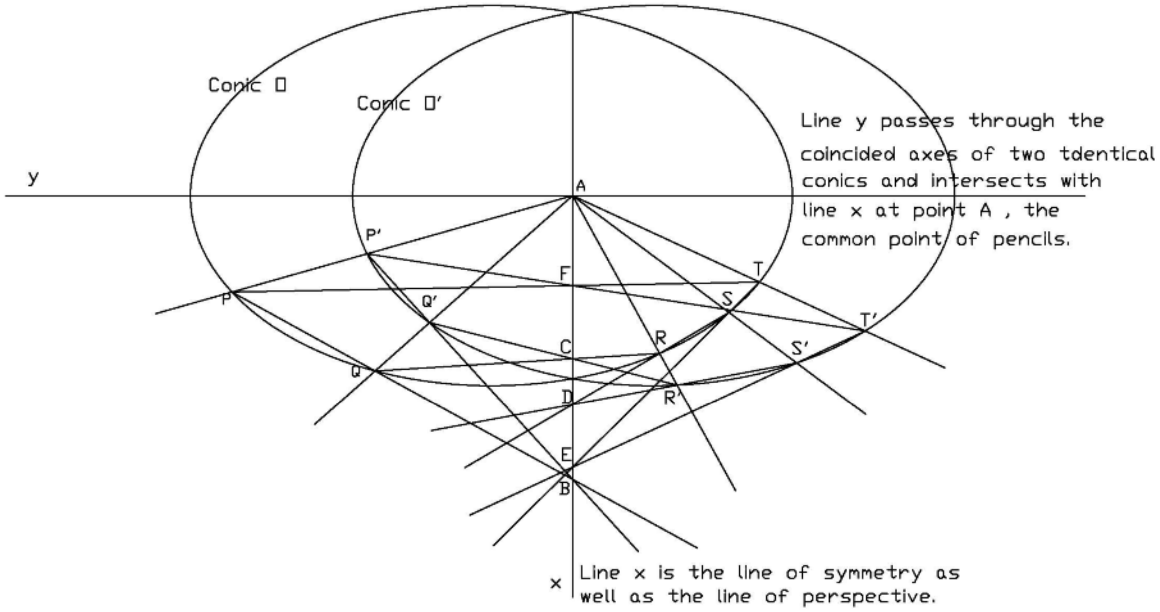


Figure 1 shows two identical conics O and O'. Line y passes through the coincided axes of the two conics and intersects with the line of symmetry of those conics at point A. Arbitrary pencils from point A intersect with those conics at various points, i.e.; points P,Q,R,S,T on the conic O and points P',Q',R',S',T' on the conic O'. The intersections B = PQ*P'Q', C = QR*Q'R', D = RS*R'S', E = ST*S'T', F = TP*T'P' are colinear with the point A and are all on the line of symmetry of the two conics (line x).

Alternative of theorem 1: Alternative of theorem 1 can be stated as follows:

The points of intersections of the corresponding sides of two polygons, each one inscribed in one of the identical and symmetrical conics, obtaining from the lines joining the intersections of pencils from a common point, which is the intersection of the line of symmetry and the line passing through the coincided axes of the two conics, and the conics lie on a straight line which is the line of symmetry.

Figure 2 shows two polygons, each one inscribed in the identical and symmetrical conics, which are perspective from a line x which is the line of symmetry of the two conics.

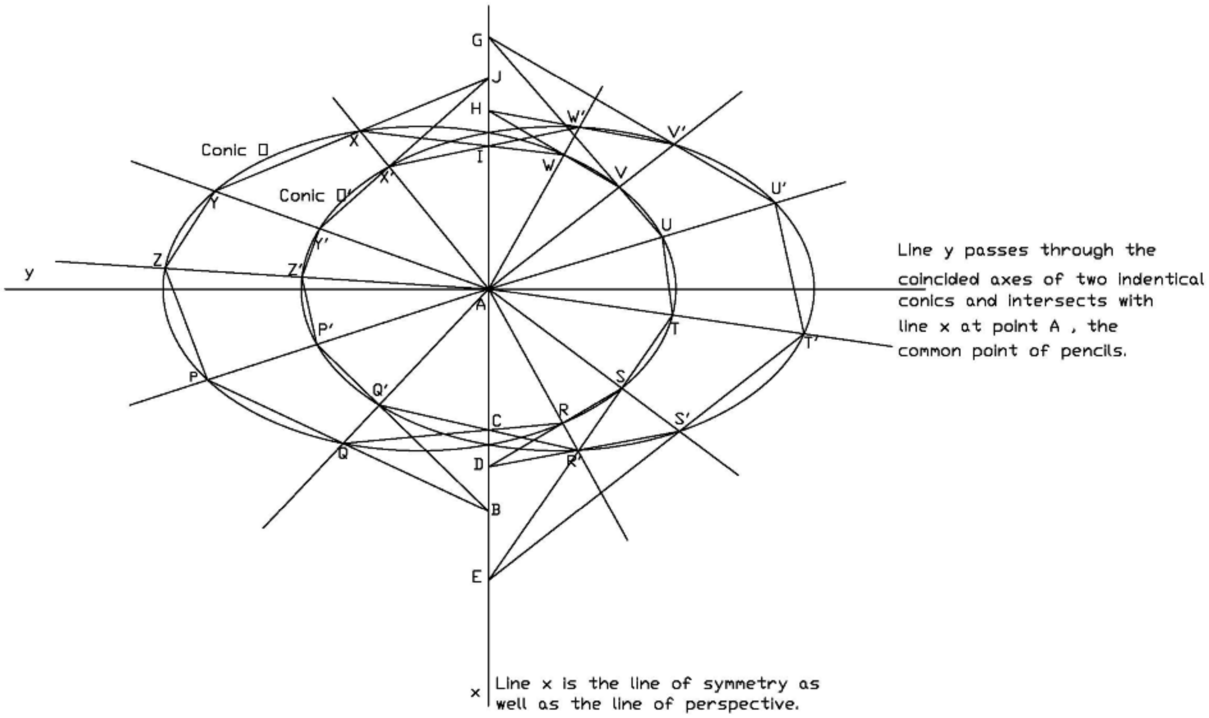


Figure 2 shows two perspective polygons PQRSTUVWXYZ and P’Q’R’S’T’U’V’W’X’Y’Z’ inscribed in two identical and symmetrical conics of which their axis are coincide. The intersections B = PQ*P’Q’, C = QR*Q’R’, D = RS*R’S’, E = ST*S’T’, G = UV*U’V’, H = VW*V’W’, I = WX*W’X’, J = XY*X’Y’ and so on lie on the line of symmetry (line x) regardless of the number and direction of pencils from A which is the intersection of line x and line y.

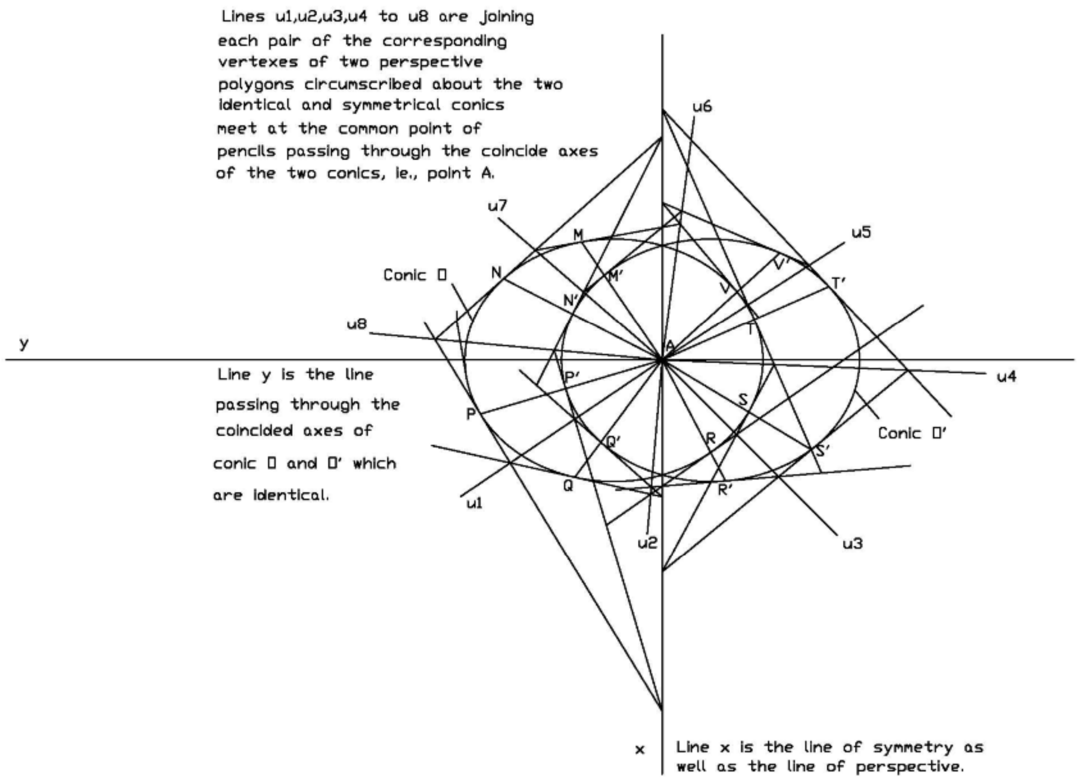
Theorem 2 (Dual of theorem 1)

To demonstrate that the intersections of tangents at the intersections of the same couple of pencils from the common point of intersection of the symmetrical line and the line passing through the coincide axes of two identical and symmetrical conics will be perspective from that point regardless of the number and direction of pencils.

Alternative of theorem 2 : Alternative of theorem 2 can be stated as follows:
The lines joining each pair of the corresponding vertexes of two polygons, each one circumscribed about one of the identical and symmetrical conics of which their axis are coincide, obtaining from the tangents at the intersections of pencils from a common point, which is the intersection of the line of symmetry and the line passing through the axis of the two conics, and the conics, pass through a point.

Figure 3 shows two polygons MNPQRSTV and M’N’P’Q’R’S’T’V’, obtaining from intersections of tangents at points M,N,P,Q,R,S,T, and V for the conic O, and points M’,N’,P’,Q’,R’,S’,T’ and V’ for the conic O’, circumscribed about the two identical and symmetrical conics ,O and O’, of which their axis are coincide, which are perspective from point A which is the common point of pencils obtaining from intersection of line x and line y regardless of the number and direction of pencils from A.

Figure 3 shows two polygons, each one circumscribed about identical and symmetrical conics which are perspective from a point A which is the common point of pencils obtaining from the intersection of the line passing through the coincided axes of the two conics and the line of symmetry of those conics.



Conclusions

Since we are free to draw any number of pencils from A in any direction as desired, therefore it is obvious that these theorems reveal the methods of constructing perspective polygons.

For example, to construct a perspective polygon of an existing polygon inscribed in a conic, just draw another identical and symmetrical conic and draw the line passing through the coincided axes of the two conics and the line passing through the intersection of the conics to get an intersection point. From this point draw pencils of the lines passing through each and every vertex of the original polygon. Then join the intersections of each pencil and the other conics and you get the perspective polygon of the original polygon on the other conic.

AutoCAD software can easily prove these two theorems and facilitate constructing of the perspective polygons governed by these theorems. This type of proving is numerical proof. Mathematicians are encouraged to prove those theorems by deductive and analytical methods.

References: *1. IMAGINATION*

2. Frank Ayres,Jr., Theory and Problems of PROJECTIVE GEOMETRY, Schaum's Outline Series, McGraw-Hill, Inc., 1967.

The Author *(Written on Apr.12, 2006)*

Sahatchai Wanawongsawad holds a BS degree in physics from Mahidol University in Bangkok. After graduation in 1978, he worked mainly in the field of water treatment with several companies. Because of illness with Major Depressive Disorder & Stress and Bipolar II Disorder, presently he does not work for any company. He has been spending his spare time as an amateur mathematician & physicist to study original theorems on Projective geometry, Calculus of non-equilibrium systems and Statistical mechanics of non-equilibrium systems since 1975. His e-mail ID is sahatchaiw@yahoo.com and website *www.sahatchaiw.com*