The Property of Similar Conics in Projective Geometry

by Sahatchai Wanawongsawad

Abstract : Similar conics are the conics of which their axis are parallel and their size and shape are proportional. Three similar conics intersect at six distinct points are demonstrated to have three lines connecting the intersections between each couple of similar conics intersect at one point. A theorem is formed from this property of similar conics. Dual of this theorem is also true for each of the three similar conics. Theorems are demonstrated and proved by AutoCAD software which is numerical proof. Proving by deductive and analytical methods are still unsolved.

Definition of Similar Conics:

Similar conics are the conics of which their axis are parallel and their size and shape are proportional.

Figure 1 and 2 are examples of three similar conics obtaining from scaling up or scaling down of an original conic while keeping their axis parallel. Both figures also show that the lines connecting the intersections between each couple of similar conics meet at one point, O. Thus, theorem 1 is formed.

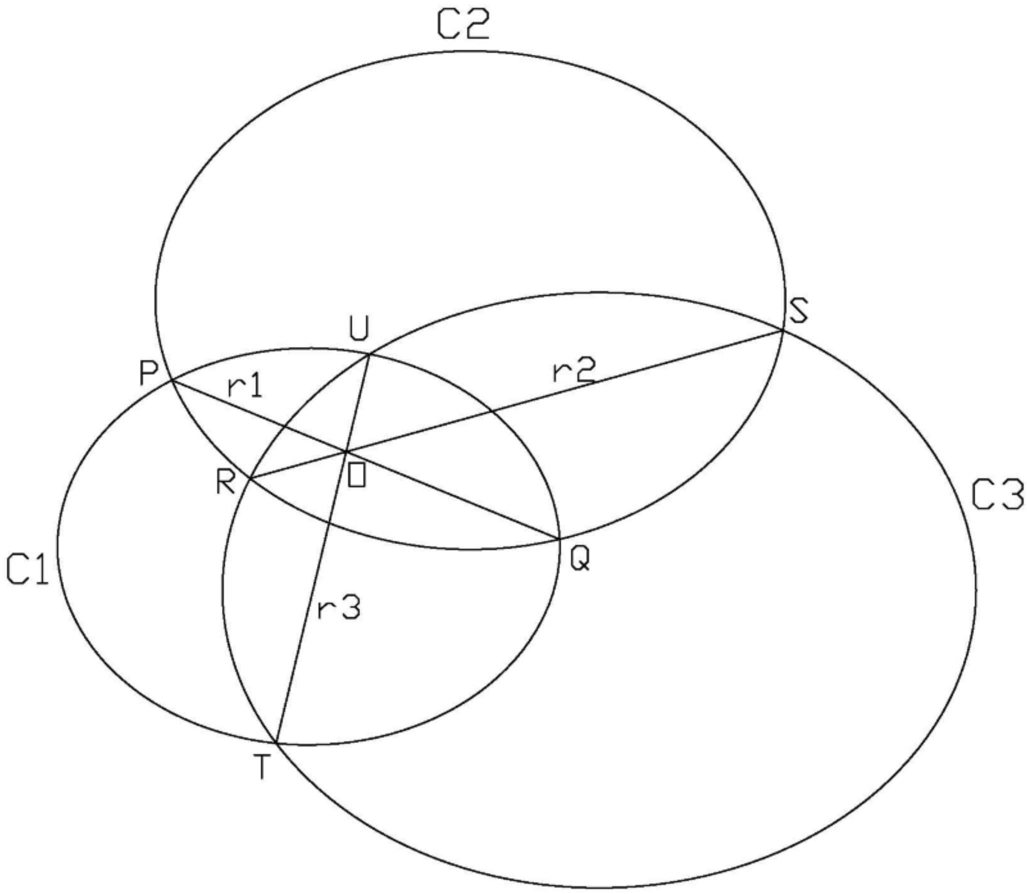


Figure 1 shows 3 similar conics C1, C2 and C3 intersect at P, Q, R, S, T, and U. The lines r1 = PQ, r2 = RS and r3 = TU intersect at one point O.

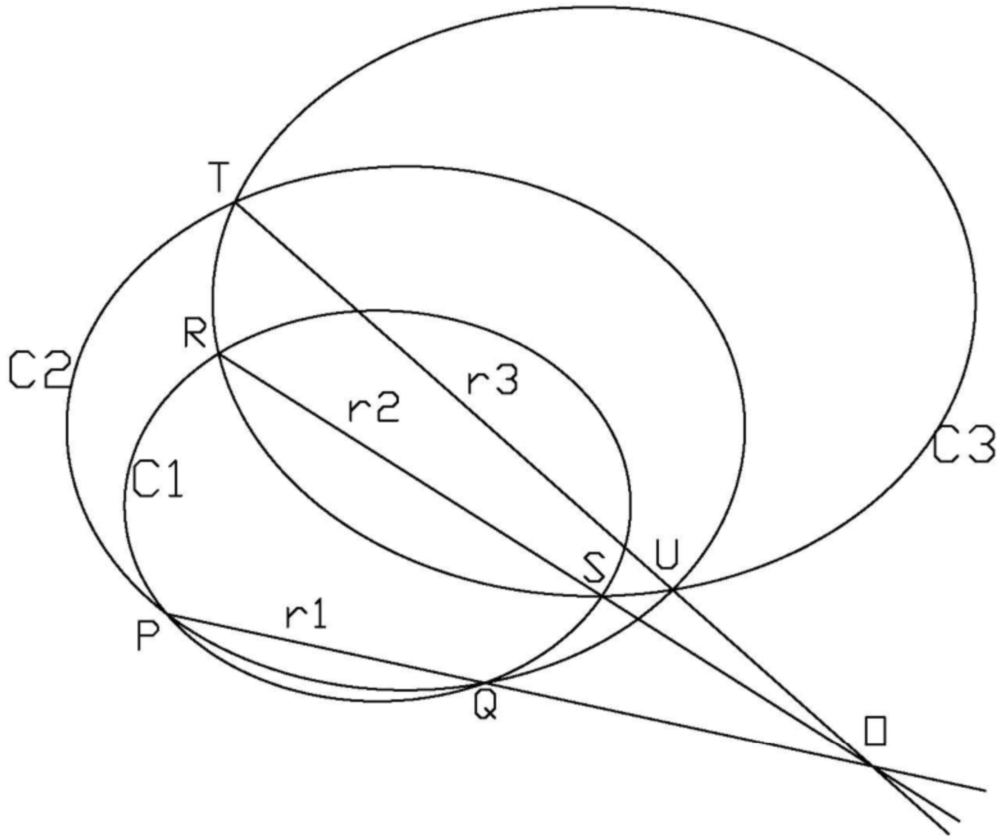


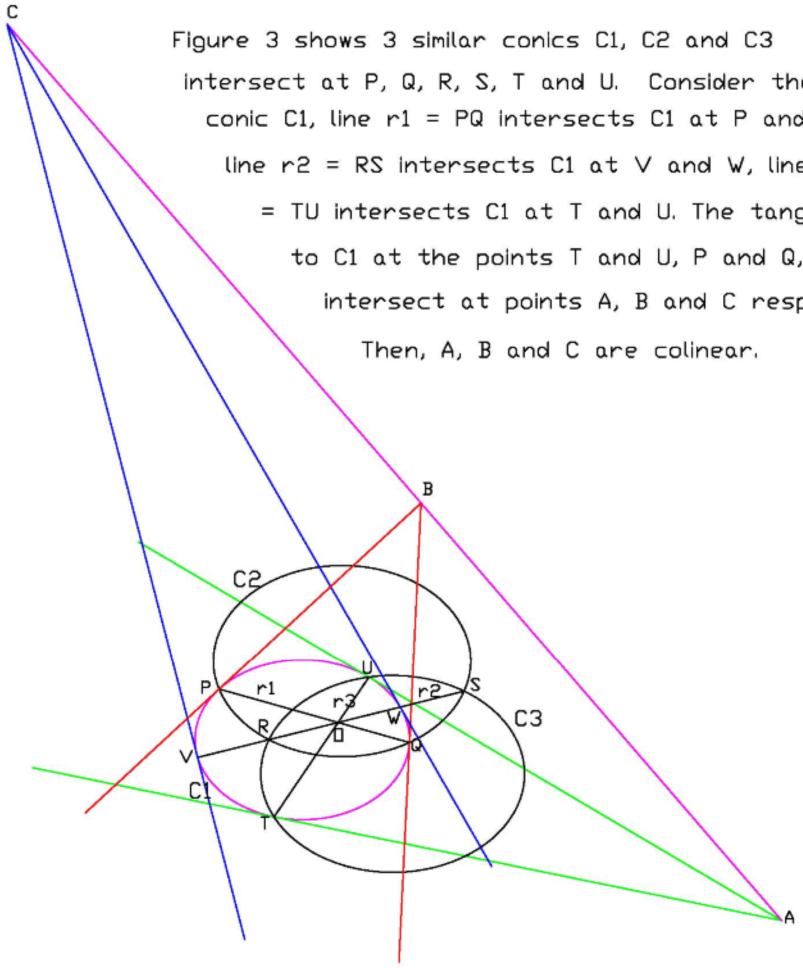
Figure 2 shows 3 similar conics C1, C2 and C3 intersect at P, Q, R, S, T and U. The lines r1 = PQ, r2 = RS, r3 = TU intersect at one point O.

Theorem 1

Three similar conics intersect at six distinct points having three lines connecting the intersections between each couple of similar conics intersect at one point.

Duality of theorem 1 are demonstrated in figure 3, 4, 5 for the group of three similar conics so aligned in such a way that the intersection point O is within the group, and in figure 6 and 7 for the group of three similar conics so aligned in such a way that the intersection point O is outside the group.

Figure 3 shows 3 similar conics C1, C2 and C3 intersect at P, Q, R, S, T and U. Consider the conic C1, line r1 = PQ intersects C1 at P and Q, line r2 = RS intersects C1 at V and W, line r3 = TU intersects C1 at T and U. The tangent lines to C1 at the points T and U, P and Q, V and W intersect at points A, B and C respectively. Then, A, B and C are colinear.



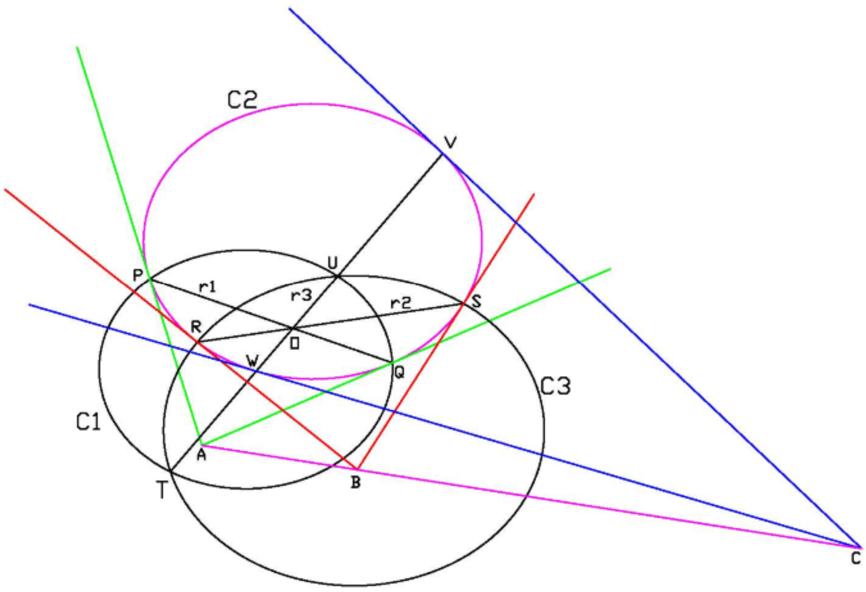


Figure 4 shows 3 similar conics C1, C2 and C3 intersect at P, Q, R, S, T and U. Consider the conic C2, line r1 = PQ intersects C2 at P and Q, line r2 = RS intersects C2 at R and S, line r3 = TU intersects C2 at V and W. The tangent lines to C2 at the points P and Q, R and S, V and W intersect at points A, B and C respectively. Then, A, B, and C are colinear.

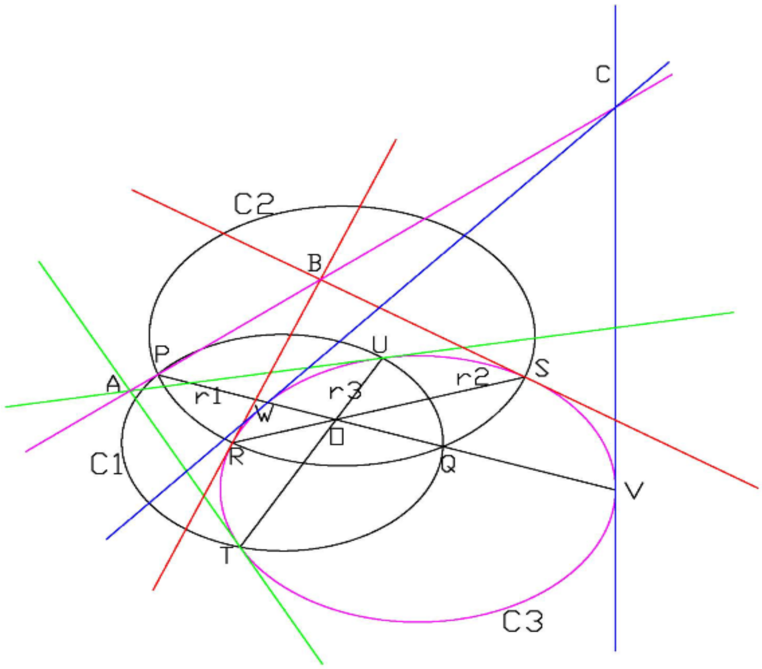


Figure 5 shows 3 similar conics C1, C2 and C3 intersect at P, Q, R, S, T and U. Consider the conic C3, line r1 = PQ intersects C3 at V and W, line r2 = RS intersects C3 at R and S, line r3 = TU intersects C3 at T and U. The tangent lines to C3 at the points T and U, R and S, V and W intersect at points A, B and C respectively. Then, A, B and C are colinear.

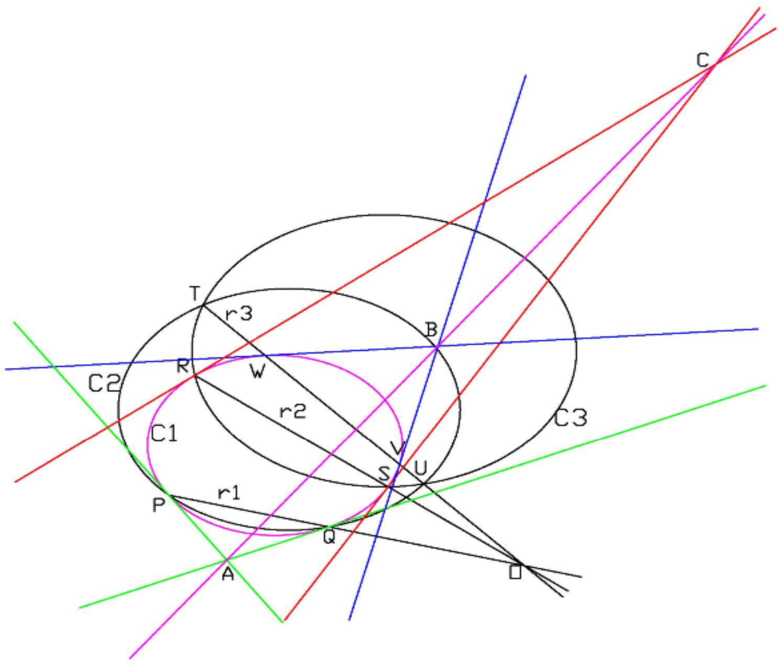


Figure 6 shows similar conics C1, C2, and C3 similar to figure 2. Consider conic C1, line r1 = PQ intersects C1 at P and Q, line r2 = RS intersects C1 at R and S, line r3 = TU interesct C1 at V and W. The tangent lines to C1 at the points P and Q, V and W, R and S intersect at points A, B and C respectively. Then, A, B, and C are colinear.

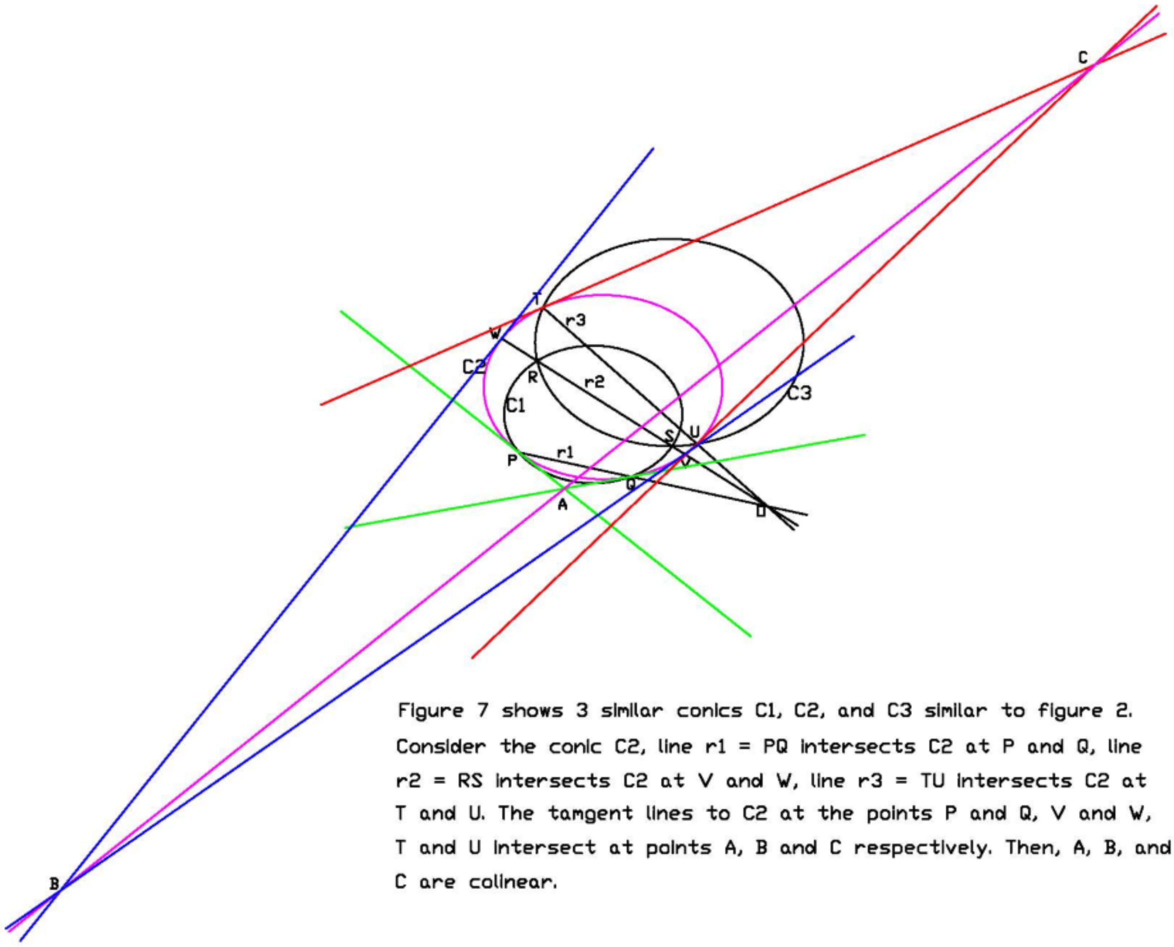


Figure 7 shows 3 similar conics C1, C2, and C3 similar to figure 2. Consider the conic C2, line r1 = PQ intersects C2 at P and Q, line r2 = RS intersects C2 at V and W, line r3 = TU intersects C2 at T and U. The tangent lines to C2 at the points P and Q, V and W, T and U intersect at points A, B and C respectively. Then, A, B, and C are collinear.

Thus, theorem 2 is formed as follows:

Theorem 2 (Dual of theorem 1)

The six tangent lines to each conic of the group of three similar conics at the intersections of each couple of conics intersect at three distinct points which are collinear.

AutoCAD software can easily prove these two theorems and facilitate constructing of the similar conics by means of scale up and down command. This type of proving is numerical proof. Mathematicians are encouraged to prove those theorems by deductive and analytical methods.

References: *1. IMAGINATION*
2. Frank Ayres,Jr., Theory and Problems of PROJECTIVE GEOMETRY, Schaum's Outline Series, McGraw-Hill, Inc., 1967.

The Author *(Written on May,1, 2006)*
Sahatchai Wanawongsawad holds a BS degree in physics from Mahidol University in Bangkok. After graduation in 1978, he worked mainly in the field of water treatment with several companies. Because of illness with Major Depressive Disorder & Stress and Bipolar II Disorder, presently he does not work for any company. He has been spending his spare time as an amateur mathematician & physicist to study original theorems on Projective geometry, Calculus of non-equilibrium systems and Statistical mechanics of non-equilibrium systems since 1975. His e-mail ID is sahatchaiw@yahoo.com and website *www.sahatchaiw.com*